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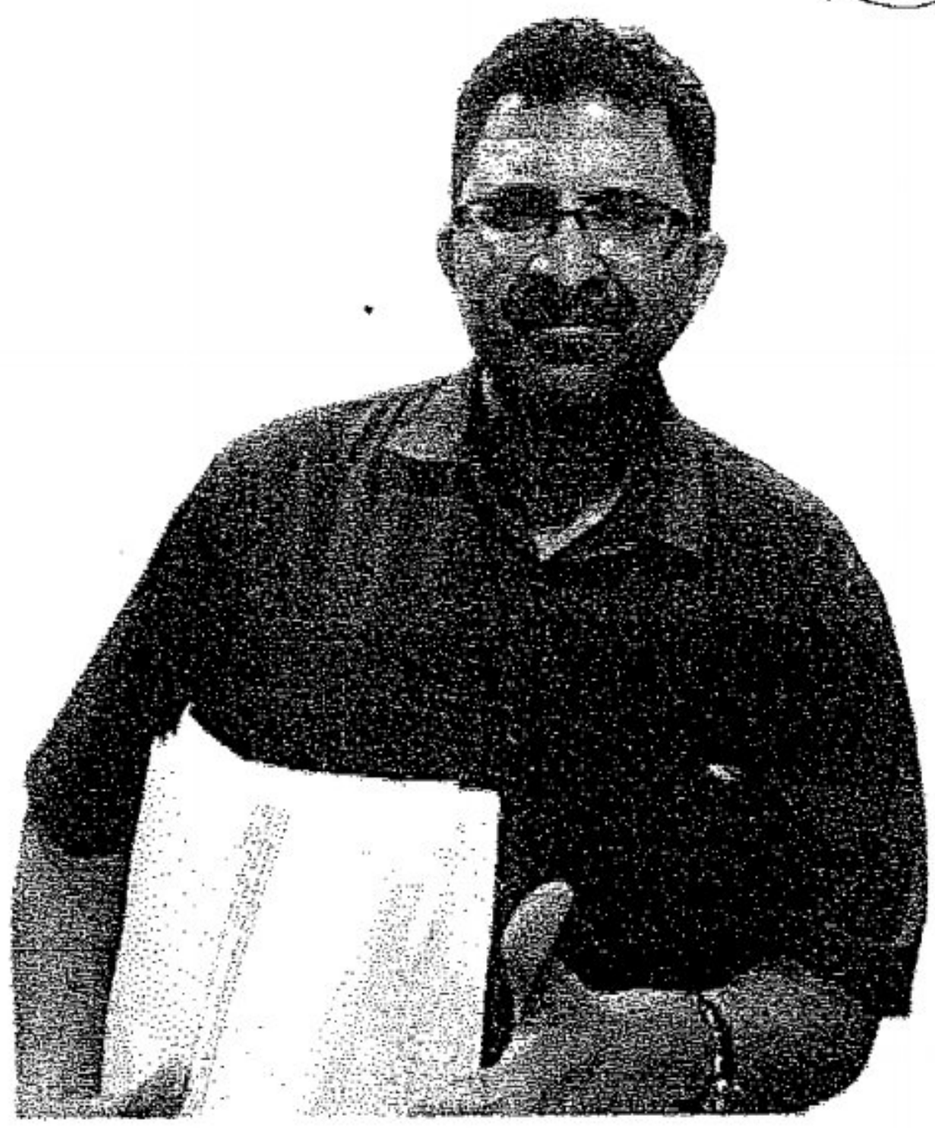
GURU ADIWIRA KEBANGSAAN 2019

ADDMATHS

TRIGONOMETRI FUNCTION
(FUNGSI TRIGONOMETRI)
PAPER 2 *(KERTAS 2)*

SPM



QUESTION 1

TRIAL
SELANGOR (SET 2)

(*a*) Tunjukkan bahawa $\dfrac{2 \text{ kosek } x}{\text{kot } x + \tan x} = 2 \text{ kos } x.$

Show that $\dfrac{2 \text{ cosec } x}{\cot x + \tan x} = 2 \cos x.$

[2 markah]
[2 *marks*]

(*b*) (i) Kemudian, lakarkan graf $y = 1 - \left| \dfrac{2 \text{ kosek } x}{\text{kot } x + \tan x} \right|$ bagi $0 \le x \le 2\pi.$

Hence, sketch the graph of $y = 1 - \left| \dfrac{2 \text{ cosec } x}{\cot x + \tan x} \right|$ *for* $0 \le x \le 2\pi.$

(ii) Seterusnya, dengan menggunakan paksi yang sama, lukis satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $2 - \left| \dfrac{2 \text{ kosek } x}{\text{kot } x + \tan x} \right| - x = 0$ untuk $0 \le x \le 2\pi.$

Nyatakan persamaan garis lurus itu dan bilangan penyelesaiannya.

Hence, using the same axes, draw a suitable straight line to find the number of solutions to the equation $2 - \left| \dfrac{2 \text{ cosec } x}{\cot x + \tan x} \right| - x = 0$, *for* $0 \le x \le 2\pi.$

State the equation of the straight line and the number of solutions.

[4 markah]
[4 *marks*]

QUESTION 2

TRIAL
SELANGOR (SET 1)

(a) Tunjukkan bahawa $\frac{2(\text{kot } x - \tan x)}{\text{sek } x \text{ kosek } x} = 2 \text{ kos } 2x$.

Show that $\frac{2(\cot x - \tan x)}{\sec x \operatorname{cosec} x} = 2 \cos 2x$.

[2 markah]
[2 *marks*]

(b) (i) Kemudian, lakarkan graf $y = \frac{2(\text{kot } x - \tan x)}{\text{sek } x \text{ kosek } x}$ bagi $0 \le x \le 2\pi$.

Hence, sketch the graph of $y = \frac{2(\cot x - \tan x)}{\sec x \operatorname{cosec} x}$ *for* $0 \le x \le 2\pi$.

(ii) Tentukan julat bagi bilangan penyelesaian, n, kepada persamaan $\frac{2(\text{kot } x - \tan x)}{\text{sek } x \text{ kosek } x} = p$, di mana p ialah satu pemalar dan $-2 \le p \le 2$.

Determine the range of the number of solution, n, for the equation $\frac{2(\cot x - \tan x)}{\sec x \operatorname{cosec} x} = p$, *where p is a constant and* $-2 \le p \le 2$.

[4 markah]
[4 *marks*]

QUESTION 3

TRIAL
NEGERI SEMBILAN

(a) Buktikan bahawa $\tan x\,(1 + \text{kos } 2x) = \sin 2x$.
Prove that $\tan x\,(1 + \cos 2x) = \sin 2x$.

[2 *markah*]
[2 *marks*]

(b) (i) Lakar graf $y = 3\sin 2x$ untuk $0 \le x \le 2\pi$.
Sketch the graph $y = 3\sin 2x$ *for* $0 \le x \le 2\pi$.

[3 *markah*]
[3 *marks*]

(ii) Seterusnya, dengan menggunakan paksi yang sama, lukis satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $6\tan x\,(1 + \text{kos } 2x) + \frac{2x}{\pi} = 4$ untuk $0 \le x \le 2\pi$.
Nyatakan bilangan penyelesaian itu.

[3 *markah*]
Hence, by using the same axes, draw a suitable straight line to find the number of solutions for equation $6\tan x(1 + \cos 2x) + \frac{2x}{\pi} = 4$ *for* $0 \le x \le 2\pi$.
State the number of solutions.

[3 *marks*]

QUESTION 4

TRIAL
NEGERI KELANTAN

(a) Lakar graf $y = -|\tan x| + 1$ untuk $0 \le x \le 2\pi$. [4 markah]

Sketch the graph of $y = -|\tan x| + 1$ *for* $0 \le x \le 2\pi$. [4 *marks*]

(b) Seterusnya, dengan melakar satu garis lurus yang sesuai pada paksi yang sama, tentukan bilangan penyelesaian yang memenuhi persamaan $-\pi(|\tan x| - 1) - \pi + x = 0$ untuk $0 \le x \le 2\pi$.

[3 markah]

Hence, by sketching a suitable straight line on the same axes, determine the number of solutions that satisfy the equation $-\pi(|\tan x| - 1) - \pi + x = 0$ for $0 \le x \le 2\pi$. [3 *marks*]

QUESTION 5

TRIAL
NEGERI PERAK

(*a*) Diberi kos $A = \frac{3}{5}$. Tanpa menggunakan kalkulator, cari nilai bagi tan $2A$.

Given $\cos A = \frac{3}{5}$. *Without using calculator, find the value of* $\tan 2A$.

[3 *markah*]
[3 *marks*]

(*b*) Lakar graf bagi $y = -2\sin\frac{3}{2}x$ bagi $0 \le x \le 2\pi$.

Sketch the graph of $y = -2\sin\frac{3}{2}x$ for $0 \le x \le 2\pi$.

Seterusnya, tentukan bilangan penyelesaiaan bagi trigonometri $x\sin\frac{3}{2}x = -\frac{\pi}{2}$ bagi $0 \le x \le 2\pi$.

[6 *markah*]

Hence, determine the number of solutions for the trigonometry $x\sin\frac{3}{2}x = -\frac{\pi}{2}$ *for* $0 \le x \le 2\pi$.

[6 *marks*]

QUESTION 6

TRIAL
SBP (ASRAMA)

(a) (i) Buktikan bahawa $\dfrac{4}{2\,\text{kot}\,\theta - 2\tan\theta} = \tan 2\theta$.

Prove that $\dfrac{4}{2\cot\theta - 2\tan\theta} = \tan 2\theta$.

[2 *markah*]
[2 *marks*]

(ii) Seterusnya, selesaikan persamaan $\dfrac{4}{2\,\text{kot}\,\theta - 2\tan\theta} = \dfrac{\sqrt{3}}{2}$ untuk $0° \le \theta \le 270°$.

Hence, solve the equation $\dfrac{4}{2\cot\theta - 2\tan\theta} = \dfrac{\sqrt{3}}{2}$ *for* $0° \le \theta \le 270°$.

[3 *markah*]
[3 *marks*]

(b) Lakarkan graf $4y + |8\sin 2x| - 8 = 0$ bagi $0 \le x \le \dfrac{3}{2}\pi$. Seterusnya, cari julat nilai k bagi persamaan $8 - |8\sin 2x| - 4k = 0$ supaya persamaan itu hanya mempunyai 6 penyelesaian untuk $0 \le x \le \dfrac{3}{2}\pi$.

Sketch the graph of $4y + |8\sin 2x| - 8 = 0$ *for* $0 \le x \le \dfrac{3}{2}\pi$. *Hence, find the range of values of* k *for the equation* $8 - |8\sin 2x| - 4k = 0$ *so that the equation has only* 6 *solutions for* $0 \le x \le \dfrac{3}{2}\pi$.

[5 *markah*]
[5 *marks*]

QUESTION 7

TRIAL
NEGERI KELANTAN

(*a*) Lakar graf bagi $y = |4\sin 2x|$ untuk $0 \le x \le \frac{3}{2}\pi$. [3 *markah*]

Sketch the graph of $y = |4\sin 2x|$ *for* $0 \le x \le \frac{3}{2}\pi$. [3 *marks*]

(*b*) Seterusnya, dengan menggunakan paksi yang sama, lakar satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $|4\sin 2x| - 1 = \frac{2x}{3\pi}$ untuk $0 \le x \le \frac{3}{2}\pi$.

Nyatakan bilangan penyelesaian itu. [3 *markah*]

Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $|4\sin 2x| - 1 = \frac{2x}{3\pi}$ *for* $0 \le x \le \frac{3}{2}\pi$.

State the number of solutions. [3 *marks*]

QUESTION 8

TRIAL
NEGERI KELANTAN

(*a*) Diberi sin m kos $n = p$ dan kos m sin $n = \frac{1}{p}$, cari nilai $\sin(m+n)\sin(m-n)$ dalam sebutan p. [2 *markah*]

Given sin m cos $n = p$ *and* cos m sin $n = \frac{1}{p}$, *find the value of* $\sin(m+n)\sin(m-n)$ *in terms of p.* [2 *marks*]

(*b*) Lakar graf $y = 1+|\tan x|$ untuk $0 \le x \le 2\pi$. [4 *markah*]

Sketch the graph of $y = 1+|\tan x|$ *for* $0 \le x \le 2\pi$. [4 *marks*]

(*c*) Seterusnya, dengan melakar satu graf yang sesuai pada paksi yang sama, tentukan bilangan penyelesaian yang memenuhi persamaan $x\left(1+|\tan x|\right)-2\pi = 0$ untuk $0 \le x \le 2\pi$. [2 *markah*]

Hence, by sketching a suitable graph on the same axes, determine the number of solutions that satisfy the equation $x\left(1+|\tan x|\right)-2\pi = 0$ *for* $0 \le x \le 2\pi$. [2 *marks*]

QUESTION 9

TRIAL
SPM 2021

(*a*) Menggunakan $\sin^2\theta + \text{kos}^2\theta = 1$, tunjukkan bahawa $\sin^4\theta + \text{kos}^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$.

[3 *markah*]

Using $\sin^2\theta + \cos^2\theta = 1$, *show that* $\sin^4\theta + \cos^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$. [3 *marks*]

(*b*) Seterusnya, selesaikan persamaan $\sin^4\theta + \text{kos}^4\theta = \frac{7}{8}$ untuk $0° \leqslant \theta \leqslant 360°$, dengan keadaan $\sin 2\theta > 0$.

[3 *markah*]

Hence, solve the equation $\sin^4\theta + \cos^4\theta = \frac{7}{8}$ *for* $0° \leqslant \theta \leqslant 360°$, *such that* $\sin 2\theta > 0$.

[3 *marks*]

QUESTION 10

TRIAL
MRSM

(a) Buktikan $\dfrac{\text{kot}^2 x}{1+\text{kot}^2 x} = \text{kos}^2 x$. [2 *markah*]

Prove that $\dfrac{\cot^2 x}{1+\cot^2 x} = \cos^2 x$. [2 *marks*]

(b) (i) Lakarkan graf $y = \dfrac{2}{\text{kos } x}\left(\dfrac{\text{kot}^2 x}{1+\text{kot}^2 x}\right)+1$ untuk $0 \le x \le 2\pi$. [3 *markah*]

Sketch the graph of $y = \dfrac{2}{\cos x}\left(\dfrac{\cot^2 x}{1+\cot^2 x}\right)+1$ for $0 \le x \le 2\pi$. [3 *marks*]

(ii) Seterusnya, dengan menggunakan paksi yang sama, lakarkan satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $\dfrac{2}{\text{kos } x}\left(\dfrac{\text{kot}^2 x}{1+\text{kot}^2 x}\right) = \dfrac{x}{\pi} - 1$ untuk $0 \le x \le 2\pi$.

Nyatakan bilangan penyelesaian itu. [3 *markah*]

Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $\dfrac{2}{\cos x}\left(\dfrac{\cot^2 x}{1+\cot^2 x}\right) = \dfrac{x}{\pi} - 1$ *for* $0 \le x \le 2\pi$.

State the number of solutions. [3 *marks*]

QUESTION 11

TRIAL
NEGERI PAHANG

(a) Lakarkan graf bagi $y = |2\cos 2x| + 1$ untuk $0 \le x \le \pi$.

Sketch the graph of $y = |2\cos 2x| + 1$ *for* $0 \le x \le \pi$.

[4 markah/*marks*]

(b) Seterusnya, dengan menggunakan paksi yang sama, lakarkan satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $|2\cos 2x| - \frac{x}{\pi} = 1$ untuk $0 \le x \le \pi$. Nyatakan bilangan penyelesaian itu.

Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $|2\cos 2x| - \frac{x}{\pi} = 1$ *for* $0 \le x \le \pi$.

State the number of solutions.

[3 markah/*marks*]

QUESTION 12

TRIAL
NEGERI JOHOR

(a) Buktikan bahawa
Prove that

$$sin^2A + kos^2A = 1$$

[2 markah/*marks*]

(b) Lakarkan graf $y = -\sin x\ ;\ 0 \leq x \leq 2\pi$.
Sketch the graph $y = -sin\, x;\ 0 \leq x \leq 2\pi$.

[2 markah/*marks*]

(c) Diberi $f(x) = a\ sinbx + c$ untuk $0° \leq x \leq 360°$. Jika amplitude bagi graf ialah 4, pusingannya ialah $120°$ dan nilai minimum bagi $f(x)$ ialah -3, nyatakan nilai a, b dan c. Seterusnya, lakarkan graf bagi fungsi tersebut.
Given $f(x) = a\ sinbx + c$ *for* $0° \leq x \leq 360°$. *If the amplitude of the graph is 4, its period is* $120°$*and the minimum value of* $f(x)$ *is* -3, *state the values of a, b and c.*
Hence, sketch the graph of the function.

[4 markah/m*arks*]

QUESTION 13

TRIAL
NEGERI TERENGGANU

(*a*) Selesaikan persamaan trigonometri $\sin(x + 30°) = 2 \text{ kos } x$ bagi $0° \le x \le 360°$.
Solve the trigonometric equation $\sin(x + 30°) = 2 \cos x$ *for* $0° \le x \le 360°$.

[3 *markah*]
[3 *marks*]

(*b*) Diberi $kos\, x = -\frac{8}{17}$ bagi $0 < x < \pi$ dan $\sin y = -\frac{24}{25}$ bagi $\frac{\pi}{2} < y < \frac{3}{2}\pi$,
tanpa menggunakan kalkulator, cari nilai

Given $\cos x = -\frac{8}{17}$ *for* $0 < x < \pi$ *and* $\sin y = -\frac{24}{25}$ *for* $\frac{\pi}{2} < y < \frac{3}{2}\pi$,
without using the calculator, find the value of

(i) sek $2x$,
sec $2x$,

[2 *markah*]
[2 *marks*]

(ii) $\sin \frac{y}{2}$.

[2 *markah*]

$\sin \frac{y}{2}$.

[2 *marks*]

QUESTION 14

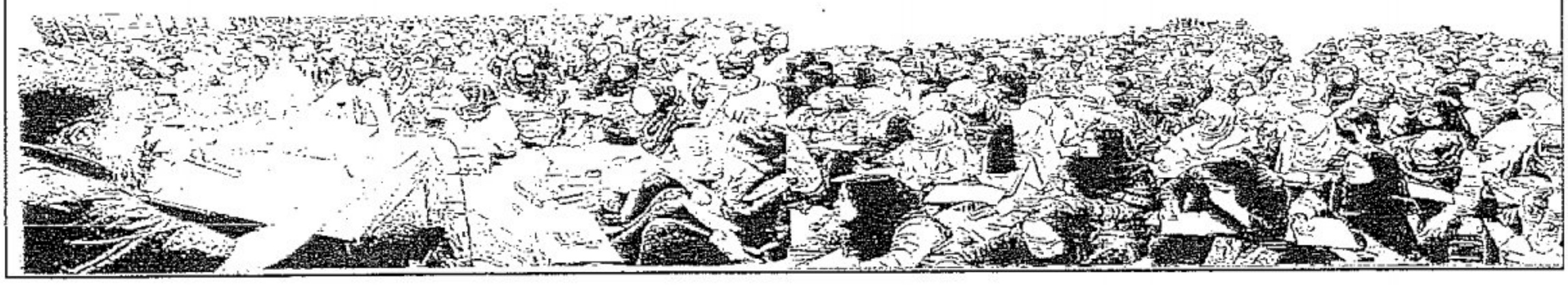
TRIAL
YIK-KELANTAN

(a) Buktikan $\sin^2\theta+\cos^2\theta=1$. [3 *markah*]

Prove $\sin^2\theta+\cos^2\theta=1$. [3 *marks*]

(b) Lakarkan graf $y=2|\sin 2x|+1$ untuk $0\le x\le 2\pi$. [4 *markah*]

Sketch the graph of $y=2|\sin 2x|+1$ *for* $0\le x\le 2\pi$. [4 *marks*]



QUESTION 15

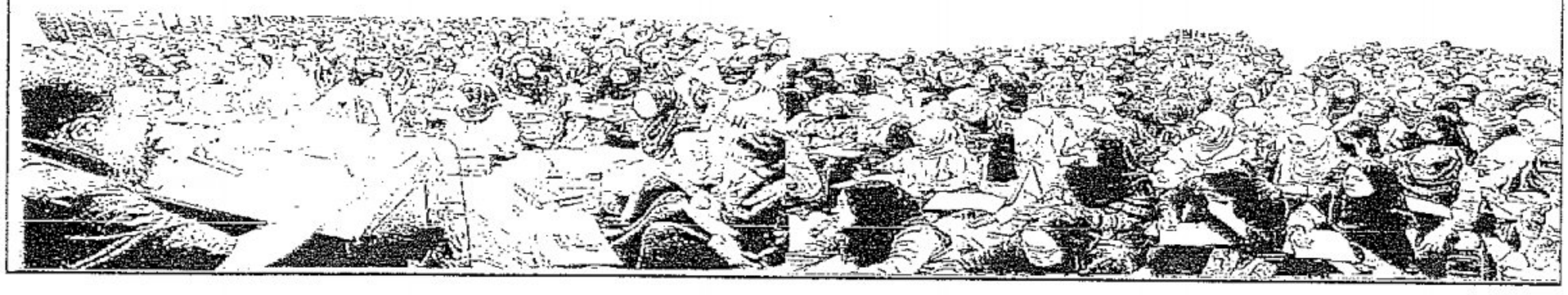
TRIAL
SABK (AGAMA)

(a) Lakarkan graf $y=\left|-\frac{3}{2}\text{kos } 2x\right|-1$ untuk $0\le x\le\frac{3}{2}\pi$. [4 *markah*]

Sketch the graph of $y=\left|-\frac{3}{2}\cos 2x\right|-1$ *for* $0\le x\le\frac{3}{2}\pi$. [4 *marks*]

(b) Seterusnya, dengan menggunakan paksi yang sama, lakar satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $3\pi\left|-\frac{3}{2}\text{kos } 2x\right|-4x=-2x$ untuk $0\le x\le\frac{3}{2}\pi$. Nyatakan bilangan penyelesaian itu. [4 *markah*]

Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $3\pi\left|-\frac{3}{2}\cos 2x\right|-4x=-2x$ *for* $0\le x\le\frac{3}{2}\pi$. *State the number of solutions.* [4 *marks*]



QUESTION 16

TRIAL
NEGERI PERLIS

(a) i) Buktikan bahawa $kosek^2x - 2sin^2x - kot^2x = kos\ 2x$

Prove that $cosec^2x - 2sin^2x - cot^2x = cos\ 2x$

[2 markah/*2 marks*]

ii) Seterusnya, selesaikan persamaan $kosek^2x - 2sin^2x - kot^2x + \frac{1}{4} = 0$

untuk $0° \leq x \leq 360°$.

Hence, solve the equation $cosec^2x - 2sin^2x - cot^2x + \frac{1}{4} = 0$

for $0° \leq x \leq 360°$

[3 markah/*3 marks*]

(b) Lakar graf bagi $y = 3kos\ 2x$ untuk $0 \leq x \leq 2\pi$.

Sketch the graph $y = 3kos\ 2x$ *for* $0 \leq x \leq 2\pi$.

[3 markah/ *3 marks*]

QUESTION 17

TRIAL
NEGERI MELAKA

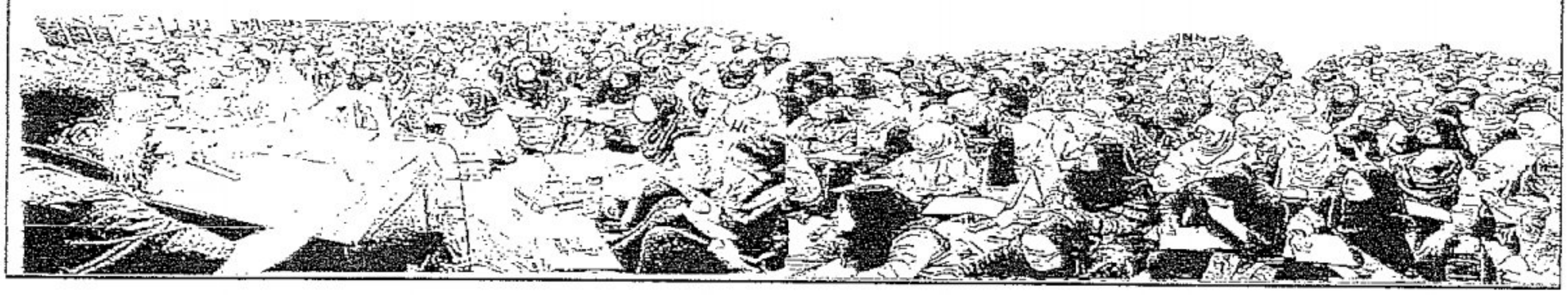
(a) Terbitkan identiti asas $sin^2x + kos^2x = 1$ [2 markah]
Derive the basic identity $sin^2x + cos^2x = 1$ [2 *marks*]

(b) (i) Lakarkan graf bagi $y = 1 - 2kos\frac{3}{2}x$ untuk $0 \leq x \leq 2\pi$ [3 markah]
Sketch the graph of $y = 1 - 2cos\frac{3}{2}x$ *for* $0 \leq x \leq 2\pi$ [3 *marks*]

(ii) Seterusnya, gunakan graf di b(i), lakar satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $\frac{5}{2} - 2cos\frac{3}{2}x - \frac{x}{\pi} = 0$ untuk $0 \leq x \leq 2\pi$. Nyatakan bilangan penyelesaian tersebut. [3 markah]

Hence, using the graph in b(i), sketch a suitable straight line to find the number of solutions to the equations $\frac{5}{2} - 2cos\frac{3}{2}x - \frac{x}{\pi} = 0$ *for* $0 \leq x \leq 2\pi$.

State the number of solutions. [3 *marks*]

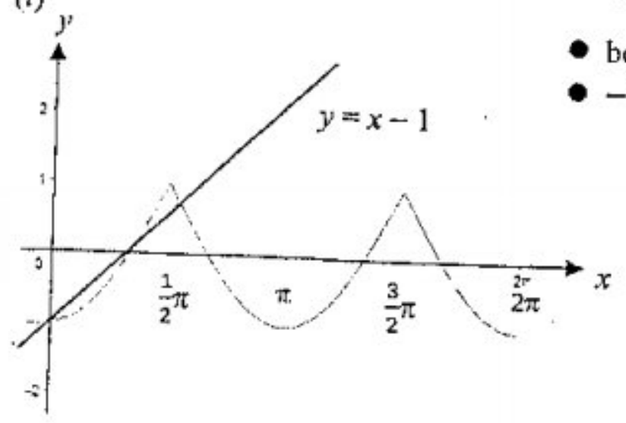


1

(a) $\dfrac{\frac{2}{\sin x}}{\frac{\kos x}{\sin x}+\frac{\sin x}{\kos x}}$

$\dfrac{2}{\sin x}\times\dfrac{\sin x\kos x}{\kos^2 x+\sin^2 x}=2\kos x$

(b) (i)



- bentuk $|\kos x|$
- $-1\le y\le 1$

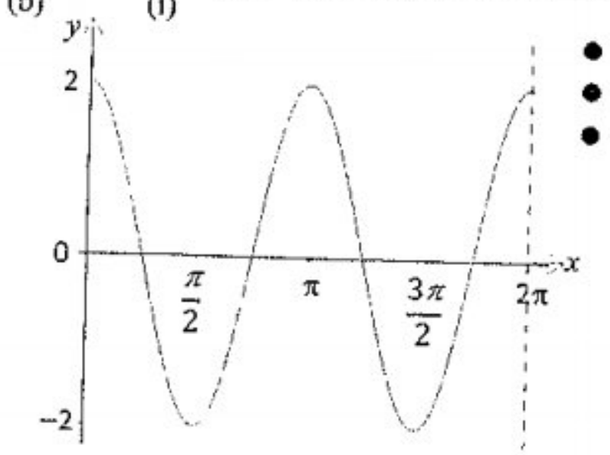
(ii) garis $y=x-1$ dilakar
3 penyelesaian

2

(a) $2\left[\dfrac{\frac{\kos x}{\sin x}-\frac{\sin x}{\kos x}}{\left(\frac{1}{\kos x}\right)\left(\frac{1}{\sin x}\right)}\right]$

$2(\kos^2 x-\sin^2 x)=2\kos 2x$

(b) (i)



- bentuk kos x
- ulang 2 kitaran
- $-2\le y\le 2$

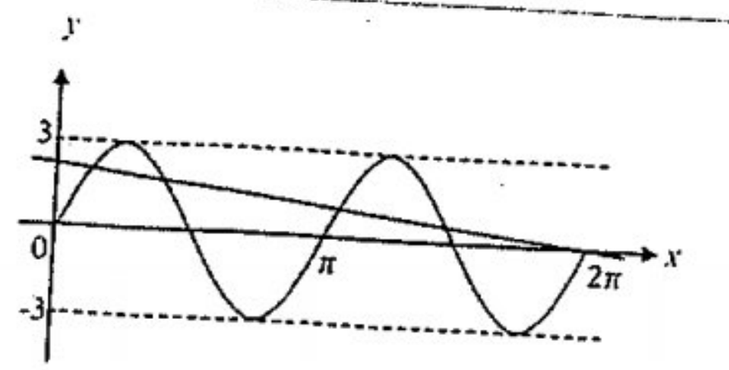
(ii) $2\le n\le 4$

3

(a) $\dfrac{\sin x}{\kos x}(1+2\kos^2 x-1)$

$\sin 2x$

(b)
(i)



Bentuk sin
Amplitud (maksimum = 3 dan minimum = −3)
2 kitaran

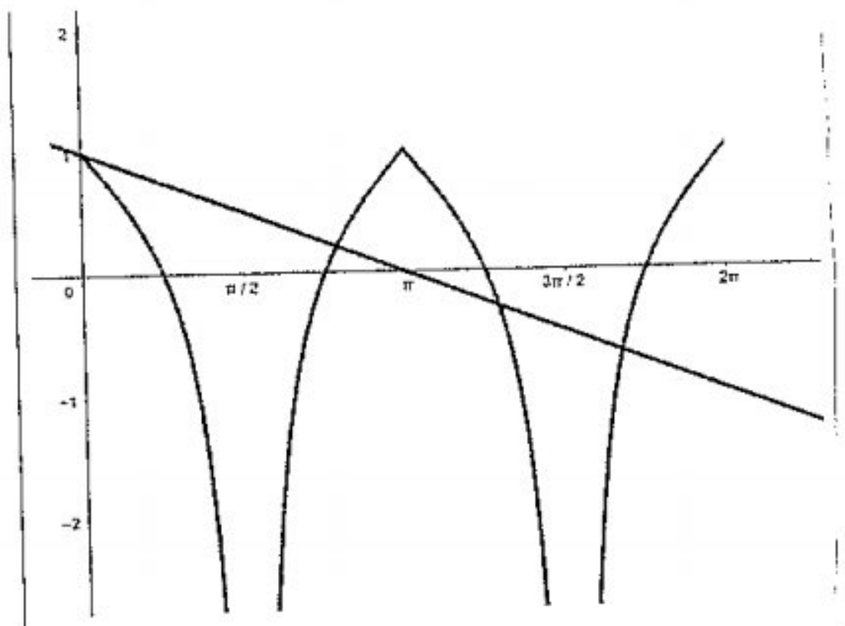
(ii) $y=2-\frac{x}{\pi}$

Graf garis lurus $y=2-\frac{x}{\pi}$

5 bilangan penyelesaian

4

(a)



Shape of tan x
1 cycle for $0\le x\le 2\pi$
Modulus graph
shifted 1

(b) $-\dfrac{x}{\pi}+1$

Graf linaer, m negatif dan pintasan $-y=1$

Bil penyelesaian = 4

5

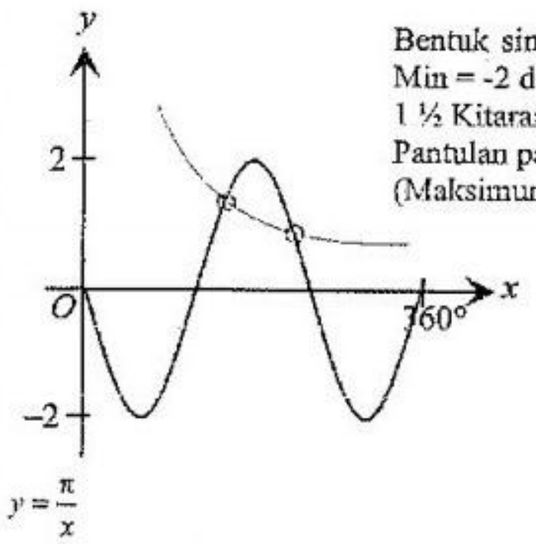
(a)

$\tan A = \frac{4}{3}$

$\frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2}$

$-3\frac{3}{7}$

(b)



Bentuk sinus
Min = -2 dan Maks. = 2
1 ½ Kitaran untuk 2π
Pantulan pada paksi-x
(Maksimum 3 markah)

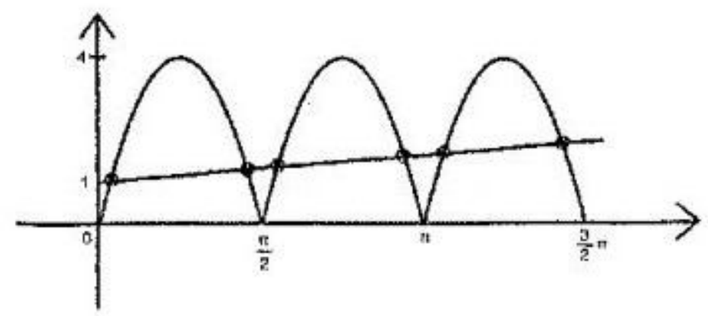
$y = \frac{\pi}{x}$

Lakar graf $y = \frac{\pi}{x}$

Bilangan penyelesaian = 2

7

(a)



(b)

Lakar garis lurus

$y = \frac{2x}{3\pi} + 1$

Bil. Penyelesaian = 6

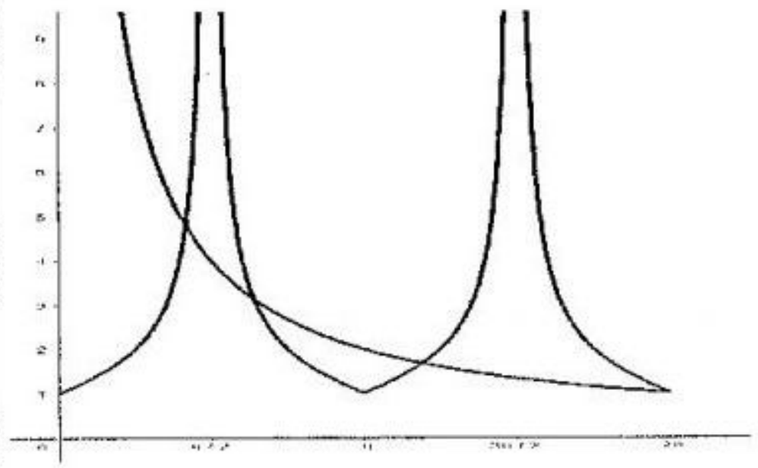
8

(a)

$\left(p+\frac{1}{p}\right)\left(p-\frac{1}{p}\right)$

$p^2 - \left(\frac{1}{p}\right)^2$ OR $\frac{p^4-1}{p^2}$

(b)



Shape of tan x

1 cycle for $0 \le x \le 2\pi$

Modulus graph

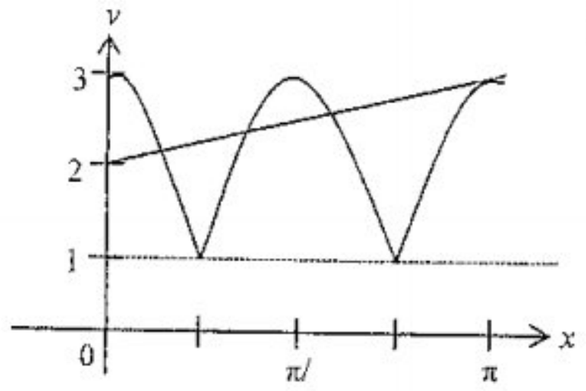
shifted 1

(c)

Graf $y = \frac{2\pi}{x}$

Bil penyelesaian = 4

11 (a)



Shape (cosine)	N1	
1 cycle	N1	
Amplitude	N1	
Modulus	N1	

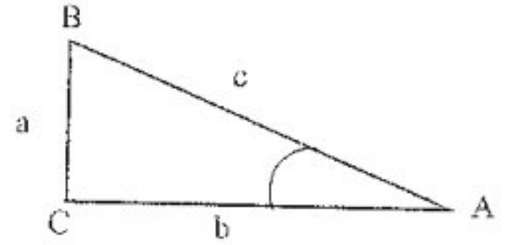
(b)

$y = 2 + \frac{x}{\pi}$ K1

Draw line $y = 2 + \frac{x}{\pi}$ N1

Number of solutions = 4 N1

12 (a)



$sinA = \frac{a}{c}$ and $kos\ A = \frac{b}{c}$ and $\tan A = \frac{a}{b}$

Teorem Pythagoras,

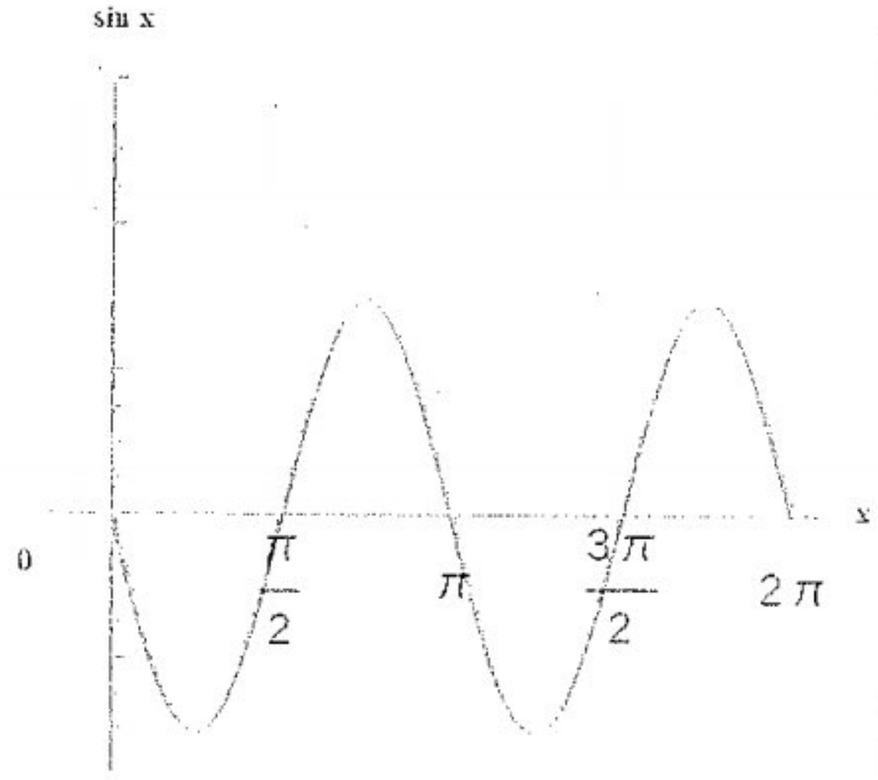
$a^2 + b^2 = c^2$ ..... ①

① ÷ c^2 : $\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

$sin^2A + kos^2A = 1$ Terbukti.

12 (b)

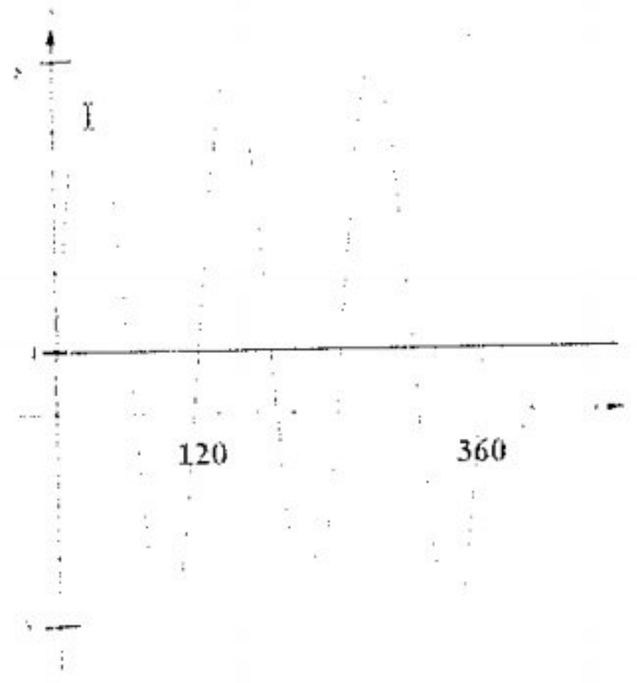


Graf bentuk sin x diberi 1 markah

Semua betul diberi 1 markah

(c) a=4, b=3,c=1

Betul semua diberi 1 markah.



Benruk graf

Amplitud

Kitaran

(13) (a) $(\sin x)\left(\frac{\sqrt{3}}{2}\right)+(\cos x)\left(\frac{1}{2}\right)=2\cos x$

$\tan x = 1.732$

$x = 60°, 240°$

(b)(i) $\frac{1}{1-2\left(\frac{15}{17}\right)^2}$

$-\frac{289}{161}$

(ii) $\pm\sqrt{\frac{1-\left(-\frac{7}{25}\right)}{2}}$

$\frac{4}{5}$

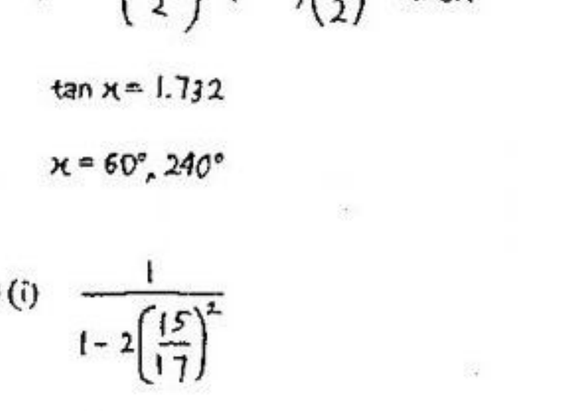
(15) (a) Graf kosinus/ *Cosinus graph*

$1\frac{1}{2}$ kitaran / *cycle*

Amplitud = $1\frac{1}{2}$ (Maksimum ; $\frac{1}{2}$, Minimum ; -1)

Mutlak **DAN** anjakan 1 unit ke bawah

*Absolute **AND** move 1 unit down*



(b) $3(y+1)-4x=-2x$

Garis lurus / *Straight line*

6 bilangan penyelesaian / *6 no of solution*

(16)

(a) (i) Guna $kosek^2x = 1 + kot^2x$ (K1)

atau

$kos\ 2x = 1 - 2sin^2x$

N1 $Kos\ 2x$

(ii) $kos\ 2x = \frac{-1}{4}$ (K1)

Sudut rujukan = 75.52° N1

$x = 52.24°, 127.76°, 232.24°, 307.76°$

(b) Graf bentuk kos

2 kalaan untuk $0 \le x \le 2\pi$

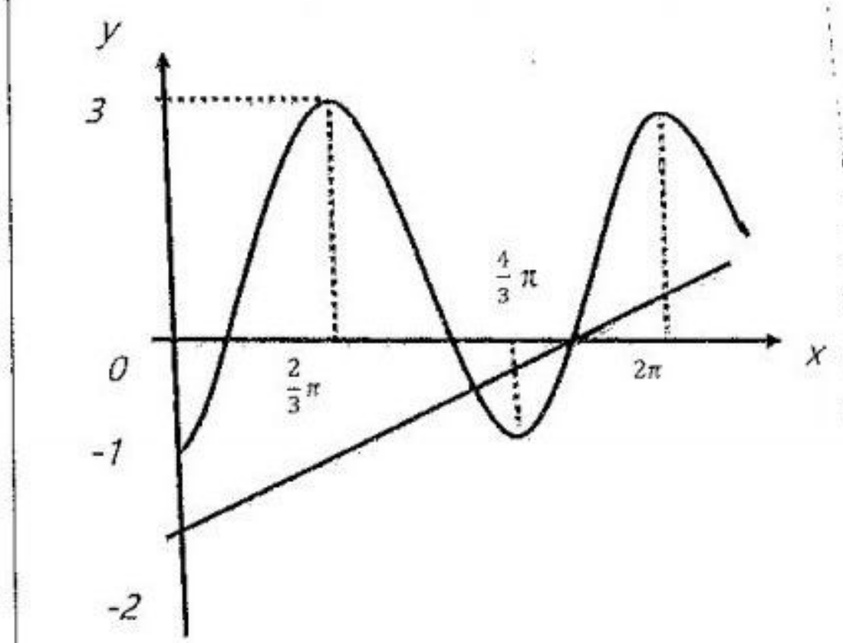
Amplitude 3

(17) (a) Gunakan $\frac{a}{c} = \sin x$ dan $\frac{b}{c} = \cos x$

dalam teorem Pythagoras $a^2 + b^2 = c^2$

$sin^2x + cos^2x = 1$

(b) i)



$y = \frac{x}{\pi} - \frac{3}{2}$

(b) ii) Sketch *straight line correctly

Number of solutions = 2